Fluids Quiz 2 Solution Fall 07

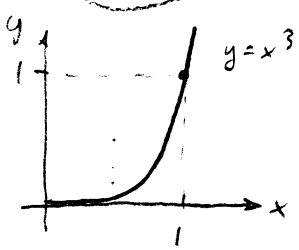
2) a) Using pathline definitions $\dot{x}_A = u$, $\dot{y}_A = v$ with $u = 1$, $v = 3xt$

$$\dot{x}_A = 1 \rightarrow \boxed{x_A(t) = t + \cancel{C}^{\,0} = t}$$ using I.C. $x_A(0) = 0$ (5 pts)

$$\dot{y}_A = 3xt = 3t^2$$

or $$\boxed{y_A(t) = t^3 + \cancel{C}^{\,0} = t^3}$$ using I.C. $y_A(0) = 0$ (10 pts)

Pathline shape $y(x) = t^3 = x^3$



b) acceleration $= \boxed{\frac{D\vec{V}}{Dt} = \frac{Du}{Dt}\hat{i} + \frac{Dv}{Dt}\hat{j}}$ at $x, y, t = (1, 1, 1)$

$$\boxed{a_x = \frac{Du}{Dt} = \cancel{\frac{\partial u}{\partial t}}^{\,0} + u\cancel{\frac{\partial u}{\partial x}}^{\,0} + v\cancel{\frac{\partial u}{\partial y}}^{\,0} = 0}$$ (10 pts)

$$\boxed{a_y = \frac{Dv}{Dt} = \cancel{\frac{\partial v}{\partial t}}^{\,3x} + u\cancel{\frac{\partial v}{\partial x}}^{\,3t} + v\cancel{\frac{\partial v}{\partial y}}^{\,0} = 3x + 1 \cdot 3t = 6}$$ (10 pts)

Alternative b) Since particle A passes through point (1, 1), we can use its pathline $x_A, y_A(t) = (t, t^3)$ to get the acceleration.

$$\vec{a} = \frac{d^2}{dt^2}\left[x_A\hat{i} + y_A\hat{j}\right] = 0\hat{i} + 6t\hat{j}$$

at $t = 1$: $\vec{a} = 0\hat{i} + 6\hat{j}$ same result